Nonlinear simulations using the BOUT++ code

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Outline

• Simulation of density fluctuations before the L-H transition for Hydrogen and Deuterium plasmas in the DIII-D tokamak using the BOUT++ code
• Non-linear simulations of the ELM triggering by the Deuterium/Hydrogen pellet injection in a divertor geometry using BOUT++ code
• Simulation of the Lithium pellet injection in a divertor geometry using the BOUT++ transport code (equations)
Simulation of density fluctuations before the L-H transition for Hydrogen and Deuterium plasmas in the DIII-D tokamak using the BOUT++ code

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Outline

- Motivation
- Linear simulation results
- Nonlinear simulation results
  - Comparison of the nonlinear spectrum between simulation and experimental results
  - Characteristic of the spectrum in linear stage simulation with $E_r$ scan
  - The ratio of the curvature related turbulence and ExB related turbulence in nonlinear stage
  - The turbulence and flux level for H plasma and D plasma
- Summary
The minimum L-H transition power threshold is different for Hydrogen (H) and Deuterium (D) plasmas
- Lower power threshold in D-plasma at lower density
- Converge at higher density

Dual modes, propagating in both ion and electron diamagnetic directions, are found when minimum L-H transition threshold power achieved in D-plasma at low density

Simulation is needed to understand the physical mechanics behind these experimental results

Yan, Z., et al. (2016). 26th IAEA, Kyoto, Japan
The experimental results can be reproduced using force balance $E_r$ with no net flow.

- Using the kinetic EFIT equilibrium and profiles from the experimental measurements
- Both the frequency and the $k_\theta$ are comparable with the experimental results

Yan, Z., et al. (2016). 26th IAEA, Kyoto, Japan
Variables in the 6-field 2-fluid model

Definitions:

\[
\sigma = n_{i0} \frac{m_i}{B_0} \left( \nabla_{\perp}^2 \phi + \frac{1}{n_{i0}} \nabla_{\perp} \phi \cdot \nabla_{\perp} n_{i0} + \frac{1}{n_{i0} Z_i e} \nabla_{\perp}^2 p_{i1} \right),
\]

\[J_\parallel = J_{\parallel0} - \frac{1}{\mu_0} B_0 \nabla_{\perp}^2 \psi,\]

\[V_{\parallel e} = V_{\parallel i} + \frac{1}{\mu_0 Z_i e n_i} \nabla_{\perp}^2 A_\parallel.\]

\[
\eta_{SP} = 0.51 \times 1.03 \times 10^{-4} Z_i \ln \Lambda T^{-3/2} \Omega \text{ m}^{-1}
\]

Flux limited expression for parallel thermal conduction:

\[
\kappa_{\parallel i} = 3.9 n_i v_{th,i}^2 / \nu_i \quad \kappa_{\parallel e} = 3.2 n_e v_{th,e}^2 / \nu_e
\]

\[
\kappa_{fs,j} = n_j v_{th,j} q R_0 \alpha_j
\]

\[
\kappa_{eff,j} = \frac{\kappa_\parallel J \kappa_{fs,j}}{\kappa_\parallel J + \kappa_{fs,j}}.
\]

---

Equations of the 6-field 2-fluid model

\[ \frac{\partial \varpi}{\partial t} = -\frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla \varpi + B_0^2 \nabla \left( \frac{J_{||}}{B_0} \right) + 2 \hat{b} \times \tilde{k} \cdot \nabla p_i \]

\[ -\frac{1}{2\Omega_i} \left[ \frac{1}{B_0} \hat{b} \times \nabla P_i \cdot \nabla (\nabla_\perp \Phi) - Z_i e B_0 \hat{b} \times \nabla n_i \cdot \nabla \left( \frac{\nabla \perp \Phi}{B_0} \right)^2 \right] \]

\[ + \frac{1}{2\Omega_i} \left[ \frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla (\nabla_\perp^2 P_i) - \nabla_\perp^2 \left( \frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla P_i \right) \right] + \mu_{||} \nabla_\parallel n_i \varpi \]

\[ \frac{\partial}{\partial t} n_i = -\frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla n_i \frac{2m_i}{Z_i e B_0} \hat{b} \times \tilde{k} \cdot \nabla \Phi - \frac{2}{Z_i e B_0} \hat{b} \times \tilde{k} \cdot \nabla P_i \]

\[ -n_i B_0 \nabla \left( \frac{V_{|| i}}{B_0} \right) \]

\[ \frac{\partial}{\partial t} V_{|| i} = \frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla V_{|| i} - \frac{1}{m_i n_i} \hat{b} \cdot \nabla P \]

\[ \frac{\partial}{\partial t} A_|| = -\nabla \Phi + \frac{\eta}{\mu_0} \nabla_\perp^2 A_|| + \frac{1}{en_0 B_0} \nabla_\parallel P_e + \frac{0.71 k_B}{e B_0} \nabla_\parallel T_e - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_|| \]

\[ \frac{\partial}{\partial t} T_i = -\frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla T_i \]

\[ -\frac{2}{3} T_i \left[ \left( \frac{2}{B_0} \hat{b} \times \tilde{k} \right) \cdot \left( \nabla \Phi + \frac{1}{Z_i e n_0} \nabla P_i + \frac{5}{2} \frac{k_B}{Z_i e} \nabla T_i \right) \right] + B_0 \nabla \left( \frac{V_{|| i}}{B_0} \right) \]

\[ + \frac{2}{3 \beta_0 k_B} \nabla_\parallel (\kappa || \nabla_\parallel T_i) + \frac{2}{3 n_i k_B} \nabla_\perp \cdot (\kappa_\perp \nabla_\perp T_i) + \frac{2m_e Z_i}{m_i \tau_e} (T_e - T_i) \]

\[ \frac{\partial}{\partial t} T_e = -\frac{1}{B_0} \hat{b} \times \nabla \Phi \cdot \nabla T_e \]

\[ -\frac{2}{3} T_e \left[ \left( \frac{2}{B_0} \hat{b} \times \tilde{k} \right) \cdot \left( \nabla \Phi - \frac{1}{en_0} \nabla P_e - \frac{5}{2} \frac{k_B}{e} \nabla T_e \right) \right] + B_0 \nabla \left( \frac{V_{|| e}}{B_0} \right) \]

\[ + 0.71 \frac{2 T_e}{3 e n_0} B_0 \nabla \left( \frac{J_{||}}{B_0} \right) + \frac{2}{3 n_i k_B} \nabla_\parallel (\kappa || \nabla_\parallel T_e) + \frac{2}{3 n_i k_B} \nabla_\perp \cdot (\kappa_\perp \nabla_\perp T_e) \]

\[ -\frac{2m_e}{m_i \tau_e} (T_e - T_i) + \frac{2}{3 n_i k_B} \eta || J^2 \]

➢ Six-field two-fluid module(\varpi, n_i, T_i, T_e, A_{||}, V_{||}): based on Braginskii equations, the density, momentum and energy of ions and electrons are described in drift ordering

Geometry used in the simulations

- ITER similar Shape, Low Torque via Balanced beam injection
- Grid resolution: 256 (radial) x 64 (poloidal)
- Radial domain: $\psi_N = 0.9 - 1.04$
- The ion density fluctuations at the middle plane ($y = 37$ and $38$) at low field side is analyzed, in order to compare with the experimental results

Yan, Z., et al. (2016). 26th IAEA, Kyoto, Japan
The profiles for H and D plasmas from DIII-D before L-H transition
The $k_\theta \rho_i$ for the maximum growth rate is similar for both H and D plasmas.

- The unstable modes for both H plasma and D plasma are resistive ballooning modes. The ideal Peeling-ballooning modes are stable.
- Lundquist number in core and vacuum region: $2 \times 10^7$ (at the separatrix) for Hydrogen plasma, and $1 \times 10^7$ for Deuterium plasma.
- The ion diamagnetic & ExB drifts have strong stabilizing effects for both H and D plasmas.
- The maximum growth rate are at $k_\theta \rho_i \sim 0.12$ or $n = 35$ for both cases.
Simulation strategy for dual modes study

• Change of the atomic number (A) for the same equilibrium profiles:
  ✓ to investigate the isotopic effects

• Scan of radial electric field (Er) by multiplying a factor:
  ✓ to investigate the ExB convection flow effects
Destabilizing effects when atom number $A$ changed to 1 using D plasma equilibrium

- Using the same D plasma equilibrium and profile
- The most unstable modes with ExB and ion diamagnetic flow is $k_\theta \rho_i \sim 0.12$
- $A = 1$ destabilizes the resistive ballooning modes
- The $k_\theta \rho_i$ for the maximum growth rate nearly no change with ion mass
- The spectrum of the growth rate v. s. mode number is broader when $A = 1$
Scan of radial electric field ($E_r$) by multiplying a factor to investigate the effects of $E_xB$ convection flow
The electron modes are related to the direction of the electric field

- The experimental results can be reproduced using force balance $E_r$ with no net flow
- The electron modes is related to the electric field: inversed $E_r$ will change the propagating direction of the electron modes

Scans of the electric field and the ion mass are needed to determine the physical mechanism of the dual-mode.

Yan, Z., et al. (2016). 26th IAEA, Kyoto, Japan
Increasing $E_r$ will induce electron modes in the linear stage of nonlinear simulation

- D plasma equilibrium and profiles used
- $A = 2$
- Only ion modes are found when force balance $E_r$ and reverse $E_r$ used
- Increasing $E_r$ will induce electron modes
- Using experimental $E_r$ ($\sim 0$) will decrease the mode frequency and wave number

BOUT++ nonlinear simulation
Curvature drift plays an important role when force balance $E_r$ is used in linear stage

- The ion diamagnetic drift will balance $ExB$ convection flow when force balance $E_r$ (with no net flow) is used
- The ion curvature drift is larger than the electron curvature drift, especially near the separatrix because $T_i > T_e$
- The velocity of the curvature drift is about 35 times smaller than the $ExB$/ion diamagnetic drift

\[
\vec{u}_\perp = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \nabla p_i}{neB^2}
\]

\[
\vec{u}_{\text{cur},i} = \frac{m_i}{2ZeB} (v_\perp^2 + 2v_\parallel^2) \hat{b} \times \hat{\kappa}
\]

\[
\vec{u}_{\text{cur},e} = -\frac{m_e}{2eB} (v_\perp^2 + 2v_\parallel^2) \hat{b} \times \hat{\kappa}
\]
The ratio of the contributions from curvature to ExB drift term is larger in D than H plasma

\[
\frac{\partial}{\partial t} T_i = -\frac{1}{B_0} b \times \nabla_{\perp} \Phi \cdot \nabla T_i - \frac{2}{3} T_i \left[ \left( \frac{2}{B_0} b \times \kappa \right) \cdot \left( \nabla \Phi + \frac{1}{Z_i e n_{i0}} \nabla P_i + \frac{5}{2} \frac{k_B}{Z_i e} \nabla T_i \right) \right] + \frac{2}{3} \eta_{||} (\kappa_{||} e \nabla || T_i) + E_{r,fb} A = 2
\]

\[
\frac{\partial}{\partial t} T_e = -\frac{1}{B_0} b \times \nabla_{\perp} \Phi \cdot \nabla T_e - \frac{2}{3} T_e \left[ \left( \frac{2}{B_0} b \times \kappa \right) \cdot \left( \nabla \Phi - \frac{1}{en_{i0}} \nabla P_e - \frac{5}{2} \frac{k_B}{e} \nabla T_i \right) \right] + \frac{2}{3} \eta_{||} (\kappa_{||} e \nabla || T_e) + E_{r,fb} A = 1 \quad (H)
\]

- The turbulences related to ExB are dominant
- The ratio of the contributions from curvature to ExB drift term is about 2 times larger in D plasma than H plasma
- Sensitive to Er
- No clear change with different ion mass
- The ratio for Te is negligibly small (~ 0.001)

The amplitude of the ion density fluctuation for H plasma is about 2.5 times larger than D plasma.

- Root mean square (RMS) of the ion density fluctuations are used to evaluate the level of the ion fluctuation.
- The level of the ion density fluctuation is about 2.5 times larger than the D plasma both in the separatrix and inner domain.
- The ion density profile is about 20% higher when A = 1 using D profiles and equilibrium.
The particle flux, ion and electron heat flux are about 3 times larger for H than D plasma

- The particle flux, ion and electron heat flux calculated by averaging over the nonlinear saturation stage after averaged over flux surface
- The larger level of the particle flux, ion and electron heat flux will makes the H plasma harder to achieve high confinement mode
- Equilibrium and profiles play more important role in particle and heat flux than isotopic
Summary

1. Linear simulations results - resistive ballooning modes
2. Characteristic of the spectrum in linear stage - ion diamagnetic flow and ExB convection flow balanced, and the curvature drift play an important role
   • Only ion modes found when force balance Er used
   • Increase Er will induce electron modes
3. Nonlinear spectrum consistent with experimental results - the dual-mode is more related with calculated force balance $E_r$ and profiles than isotopic
   • Fluctuations associated with ExB convection are dominant (~10 times larger than the fluctuations induced by ion diamagnetic and curvature drift in the D case and ~20 times larger in H cases)
   • The ratio of the curvature terms will increase when the direction Er reverses and will decrease when force balance Er is multiplied by 2
   • The ion mode ratio shows little correlation with ion mass scan
4. The particle flux, electron and ion heat flux are larger in H plasma - harder to achieve H mode
Non-linear simulations of the ELM triggering by the Deuterium/Hydrogen pellet injection in a divertor geometry using BOUT++ code

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- Introduction and methods
- BOUT++ five-field 2-fluid model
- Simulation methods
  - Scan of pellet parameters
    - ELM can be triggered when pellet size larger than a threshold
    - Larger ELMs are triggered when the pellet locates at the separatrix
  - Separately include the pellet perturbations in plasma density and temperature profiles and $E_r$ profile
    - Large ELMs are found when pellet included in the $E_r$ only
  - Perpendicular heat diffusion induces larger ELMs
- Summary
Pellet injection is one of the most promising methods to control the edge localized modes (ELMs)

ELM pacing has been successfully demonstrated on ASDEX, JET, DIII-D and EAST tokamaks

Larger pellet size and higher pellet speed are needed to trigger ELMs

The divertor heat flux can be mitigated by pellet

Both H and Li pellet have been used in pacing ELMs

P. T. Lang et al, Nucl. Fusion 53(2013) 073010
L. R. Baylor et al, VLT Conference (2013)
ELMs were successfully paced using lithium granule injector (LGI) on EAST

The amplitude is lower than the natural ELMs with frequency controlled by lithium pellet

Simulations are needed to optimize the pellet injection parameters
• A critical value of the pellet size is needed to trigger the ELM
• The larger pellet leads to larger heat flux in the divertor
• ELM triggered by the ballooning instabilities caused by pellet ablation induced local density and temperature perturbation

S. Futatani et al, 24th ITPA-PEP, (2016)
Equations of the simplified 6-field 2-fluid model of BOUT++ code

\[
\frac{\partial \varpi}{\partial t} = -\frac{1}{B_0} \hat{b} \times \nabla_\perp \varpi + B_0^2 \nabla_\parallel \left( \frac{J_\parallel}{B_0} \right) + 2\hat{b} \times \hat{k} \cdot \nabla p_i \\
- \frac{1}{2\Omega_i} \left[ \frac{1}{B_0} \hat{b} \times \nabla P_i \cdot \nabla \left( \nabla_\perp^2 \varpi \right) - Z_i e B_0 \hat{b} \times \nabla n_i \cdot \nabla \left( \frac{\nabla_\perp \varpi}{B_0} \right) \right] \\
+ \frac{1}{2\Omega_i} \left[ \frac{1}{B_0} \hat{b} \times \nabla \varpi \cdot \nabla \left( \nabla_\parallel^2 P_i \right) - \nabla_\parallel \left( \frac{1}{B_0} \hat{b} \times \nabla \varpi \cdot \nabla P_i \right) \right] + \mu_\parallel i \nabla_\parallel^2 \varpi
\]

\[
\frac{\partial n_i}{\partial t} = -\frac{1}{B_0} \hat{b} \times \nabla_\perp \varpi \cdot \nabla n_i
\]

\[
\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \phi - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_\parallel
\]

\[
\frac{\partial T_i}{\partial t} = -\frac{1}{B_0} \hat{b} \times \nabla_\perp \varpi \cdot \nabla T_i + \frac{2}{3n_i 0 k_B} \nabla_\parallel \left( \kappa_i \nabla_\parallel 0 T_i \right)
\]

\[
\frac{\partial T_e}{\partial t} = -\frac{1}{B_0} \hat{b} \times \nabla_\perp \varpi \cdot \nabla T_e + \frac{2}{3n_e 0 k_B} \nabla_\parallel \left( \kappa_e \nabla_\parallel 0 T_e \right)
\]

- Only keep the gyro-viscosity and parallel thermal conduction terms in the 6-field 2-fluid model


➢ Five-field two-fluid module(\(\varpi, n_i, T_i, T_e, A_\parallel\)): based on Braginskii equations, the density, momentum and energy of ions and electrons are described in drift ordering.

<table>
<thead>
<tr>
<th>Gyro-viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conduction</td>
</tr>
</tbody>
</table>

**Inequations:**

- \(J_\parallel\) and \(\hat{b}\) are determined by \(\frac{\partial \varpi}{\partial t}\) according to self-consistency.
DIII-D geometry and profile

- Use kinetic EFIT equilibrium to generate the grids
- Grid resolution: 256(radial) x 64(poloidal)
- Radial domain: $\psi_N = 0.85 - 1.1$
- The ideal peeling-ballooning mode is stable
Assumptions: Pellet ablating at one location and fully ionized

- Pellet motion: ~ 100 m/s
- MHD instabilities: 1-100 kHz
- The expansion, ionization and charge exchange processes are faster

The pellet density perturbation is simplified to a fully ionized Gaussian-shaped plasma at one location

\[

\nu_I = n_e \langle \sigma_I V_{th,e} \rangle = n_e c m^{-3} \cdot 3 \times 10^{-8} (0.1 T_e eV)^2 /[3 + (0.1 T_e eV)^2] s^{-1}
\]

\[

\nu_{CX} = n_i \langle \sigma_{CX} V_{th,i} \rangle = 1.7 \times 10^{-8} n_i c m^{-3} + 1.9 \times 10^{-8} n_i c m^{-3}
\]

\[\times \frac{(1.5 T_i eV)^{1/3} - (15 eV)^{1/3}}{(150 T_i eV)^{1/3} - (15 eV)^{1/3}} s^{-1}\]

Induce an Gaussian shape pellet in density profile keeping the pressure profile unchanged

- The total pressure is kept unchanged:
  - Assuming pellet only provide the particle source not the energy source
  - Keep the same MHD equilibrium during pellet parameter scan
- A Gaussian shaped pellet is induced enabling the pellet parameters scan

\[ n_{plt}(\psi, \theta) = A_{plt} \exp \left(-\left(\frac{(\psi - \psi_0)^2}{2\sigma_\psi^2} + \frac{(\theta - \theta_0)^2}{2\sigma_\theta^2}\right)\right) \]
Conducting pellet size scan by increasing the amplitude coefficient in the pellet function
Keep the pellet in the peak gradient region
Fix the radial and poloidal shape of the pellet
Scan of the pellet size by changing the pellet amplitude

- A strong local density gradient at the pellet location
- The radial electric field is changed by the pellet

The density gradient length:
\[ L_{n_i} = -\frac{n_i}{\nabla n_i} \]

The ion diamagnetic Er:
\[ E_{r,0} = \frac{\nabla P_{i,0}}{Z_i n_i e} \]
ELMs can only be triggered when the pellet size exceeds a critical value.

- The ELM will be triggered when pellet size exceeds certain value.
- Clear pressure pedestal collapse when ELM triggered.

$$\Delta_{ELM}(t) = \frac{\int_{in}^{sep} J d\psi \int d\theta d\zeta (P_0 - \langle P(t) \zeta \rangle)}{\int_{in}^{sep} J d\psi \int d\theta d\zeta}$$
Dominant n=0 modes are found when large pellet is included.

- Dominant low-n modes can enhance the phase coherent time and trigger larger ELM.
- Agree with the experiments: low frequency modes are found when pellets are injected to the plasma.
The total power across separatrix is larger when ELM trigged

The average heat flux across separatrix in the nonlinear saturation stage:

\[
\bar{W} = \frac{1}{T} \int \int d\theta d\zeta \int dt Q_r
\]

\[
Q_{jr} = \langle p_j V_r \rangle = \left\langle n_j T_j \frac{(\hat{b}_0 \times \nabla \Phi)_r}{B_0} \right\rangle
\]

- The time averaged power across separatrix increases significantly when the pellet size is larger than the critical value
Scan of the pellet injection position by changing the radial location of the Gaussian function

- Different pellet radial positions located at the pedestal top, peak gradient region and separatrix are used
- Same size and shape are used in the simulation
- The density gradient induced by the pellet are much larger when pellet located at the separatrix
The pellet at the separatrix will trigger larger ELMs

- The ELMs are larger when the pellet is at the separatrix
- The pellet located at the pedestal top only change the pressure locally
The \( E_r \) (shear) are changed by the ablated plasma from pellet.

- The radial electric fields are changed by the ablated plasma, which can change the equilibrium \( \text{ExB} \) flow shear.

\[
A_{\text{pit}} = 0.4 \times 10^{19} \text{ m}^{-3}
\]
Larger ELMs triggered with pellet perturbations in $E_r$

- The ELM size is larger when the pellet is located at the separatrix.
- Larger ELM is triggered when the ablated plasma considered in $E_r$. 
- The density turbulence will be initialized at the pellet position.
- The radial expansion of the turbulence is more obvious when the pellet considered in the $E_r$. 

\[ n_{i,RMS}, A_{plt} = 0.4 \times 10^{19}\text{m}^{-3} \]
Larger growth rate outside the separatrix with pellet perturbations in $E_r$

The growth rate of the density fluctuation:

$$\gamma = \gamma_n + \gamma_v + \gamma_{NL}$$

$$= \vec{V}_{E_1 \times B} \cdot \nabla n_i + \vec{V}_{E_0 \times B} \cdot \nabla n_i + \vec{V}_{E_1 \times B} \cdot \nabla n_i$$

- Initial growth rates are related more to the density gradient
- The radial spreading of the density fluctuations related more to the ExB flow and nonlinear interaction between flow and density fluctuation
The heat diffusion terms:

\[ Q_{||j} = \frac{2}{3n_j} \nabla_{\parallel 0} (\kappa_{||j} \nabla_{\parallel 0} T_j), \]
\[ Q_{\perp j} = \frac{2}{3n_j} \nabla_{\perp 0} (\kappa_{\perp j} \nabla_{\perp 0} T_j), \]

- With the parallel heat diffusion considered, the ELM size will be stabilized.
- With the perpendicular heat diffusion considered, the ELM size will be destabilized.
Scan of pellet parameters

- ELM can be triggered when pellet size larger than a threshold
- Larger ELMs are triggered when the pellet induced density perturbations locates at the separatrix

Separately include the pellet in plasma density and temperature profiles and $E_r$ profile

- Large ELMs are found when pellet perturbations considered in the $E_r$ only

Parallel heat diffusion will stabilize the ELM while the perpendicular heat diffusion will destabilize the ELM
Simulation of the Lithium pellet injection in a divertor geometry using the BOUT++ transport code

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Equations for background plasmas ions in the BOUT++ code

- Equations obtained by reduction of the Bragniskii equations interaction with Lithium ions considered
- Variables: ion density ($N_i$), ion temperature ($T_i$), ion parallel velocity ($V_{||,i}$) and electron temperature ($T_e$)

\[
\begin{align*}
\frac{\partial N_i}{\partial t} &= -\nabla (V_{||,i} N_i) + D_{\perp,i} \nabla^2 N_i \\
\frac{\partial T_i}{\partial t} &= \frac{2}{3N_i} \nabla_{||} \left( \kappa_{||,e} \nabla_{||} T_e \right) + \frac{2}{3} \chi_{\perp,e} \nabla^2 T_e - \frac{2m_e}{M_i} \frac{T_e - T_i}{\tau_e} \\
&\quad - \left( T_e + \frac{2}{3} W_{Li^{n+}} \right) - \sum_{i=1}^{n} \frac{2m_e}{ML_i} \frac{T_e - T_{Li^{n+}}}{\tau_{e,Li^{n+}}} \\
\frac{\partial T_e}{\partial t} &= -V_{||,i} \nabla_{||} T_i - \frac{2}{3} T_i \nabla_{||} V_{||,i} + \frac{2}{3N_i} \nabla_{||} \left( \kappa_{||,i} \nabla_{||} T_i \right) + \frac{2}{3} \chi_{\perp,i} \nabla^2 T_i \\
&\quad + \frac{N_e}{N_i} \frac{2m_e}{M_i} \frac{T_e - T_i}{\tau_e} + \sum_{i=1}^{n} \frac{2M_i}{ML_i} \frac{T_{Li^{n+}} - T_i}{\tau_{i,Li^{n+}}} \\
\frac{\partial V_{||,i}}{\partial t} &= -V_{||,i} \nabla_{||} V_{||,i} + \frac{4}{3N_iM_i} \nabla_{||} \left( \eta_i \nabla_{||} V_{||,i} \right) - \frac{\nabla_{||} P_i}{N_iM_i} - \sum_{i=1}^{n} F_i
\end{align*}
\]

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Equations for Lithium ions in the BOUT++ code

- The equations for the lithium ion density ($N_{Li^{n+}}$), ion temperature ($T_{Li^{n+}}$), ion parallel velocity ($V_{||,Li^{n+}}$) are similar with background ions.
- Ionization, charge exchange and recombination are considered as source and sink terms in lithium ion density equations.

\[
\frac{\partial N_{Li^{n+}}}{\partial t} = -\nabla_{||} (V_{||,Li^{n+}} N_{Li^{n+}}) + D_{||,Li^{n+}}^c \nabla^2 N_{Li^{n+}} + S_{I^{n+}}^{Li^{n+}} - S_{rec}^{Li^{n+}} + S_{CX}^{Li^{n+}}
\]

\[
\frac{\partial V_{||,Li^{n+}}}{\partial t} = -V_{||,Li^{n+}} \nabla_{||} V_{||,Li^{n+}} + \frac{4}{3N_{Li^{n+}} M_{Li^{n+}}} \nabla (\eta_{Li^{n+}}^0 \nabla_{||} V_{||,Li^{n+}}) - \frac{\nabla_{||} P_{Li^{n+}}}{N_{Li^{n+}} M_{Li^{n+}}}
\]}

\[
- \frac{N_e}{N_{Li^{n+}}} \nu_{I^{n+}} V_{||,Li^{n+}} + \frac{N_i M_i}{N_{Li^{n+}} M_{Li^{n+}}} F_i
\]

\[
\frac{\partial T_{Li^{n+}}}{\partial t} = -V_{||,Li^{n+}} \nabla_{||} T_{Li^{n+}} - \frac{2}{3} T_{Li^{n+}} \nabla_{||} V_{||,Li^{n+}} + \frac{2}{3N_{Li^{n+}}} \nabla_{||} (k_c^c_{||,Li^{n+}}) \nabla_{||} T_{Li^{n+}}
\]

\[
+ \frac{2}{3} \chi_{||,Li^{n+}} \nabla^2 T_{Li^{n+}} + \frac{N_e}{N_{Li^{n+}}} (\nu_{rec}^{Li^{n+}} - \nu_{I^{n+}}^{Li^{n+}}) T_{Li^{n+}} + \frac{2m_e}{m_i} \frac{T_e - T_{Li^{n+}}}{\tau_{e^{Li^{n+}}}}
\]}

\[
+ \frac{2M_i N_i}{N_{Li^{n+}} M_{Li^{n+}}} \frac{T_i - T_{Li^{n+}}}{\tau_{i,Li^{n+}}}
\]
Equations for Lithium atoms in BOUT++ code

- Uniform expansion of the Lithium atom at the sound speed is assumed
- The ionization, charge-exchange and recombination rate of the Lithium ions are from ADAS database

\[
\frac{\partial N_{Li}^a}{\partial t} + \nabla \cdot (c_s N_{Li}^a) = -S_{I}^{Li^{n+}} - S_{CX}^{Li^{n+}} + S_{rec}^{Li^{n+}}
\]

\[n_e = 1.0 \times 10^{19} \text{ m}^{-3}\]
Model for the small pellet ablation

• **Ablation models**
  • Spherical symmetric model
  • The value of $f_s$ (empirical)
  • **Shielding mechanisms/size/shape dependent**
  • Gas/plasma dynamic (NGPS)
  • Magnetic shielding
  • Electrostatic shielding ($\Delta \Phi = 1-2 k_B T_e/e$)
  • Deviations from spherical symmetry
  • Modeling with more sophisticated dust codes

\[
\frac{dN}{dt} = \frac{4\pi r^2 q_\infty f_s}{\Delta E}
\]

*Parks et al; Milora et al; Wang et al, RSI (2003), JPP (2016)*